No iodine or bromine could be detected in the small quantity of water available for examination.

Spectroscopic examination revealed the presence of a trace of barium. The quantity present would have been quite unweighable, and although estimated with the calcium could not have vitiated the results.

The results given under the heading B are regarded as those which most nearly represent the true condition of the lake, and consequently no attempt has been made to strike an average between the two series of results. The A results are given in extenso in order to demonstrate the degree of reliability of the B results, a matter which will be of importance in the future, when, after an interval of some years, another investigation of the water of Lake Urmi is made.

"On the Application of Fourier's Double Integrals to Optical Problems." By CHARLES GODFREY, B.A., Scholar of Trinity College, Isaac Newton Student in the University of Cambridge. Communicated by Professor J. J. THOMSON, F.R.S. Received June 12,—Read June 15, 1899.

(Abstract.)

The propagation of plane plane-polarised light in the direction of z is governed by the equation $V^2\frac{\delta^2\phi}{\delta z^2} = \frac{\delta^2\phi}{\delta t^2}$, where V is the velocity of light.

The most simple solution of this equation is—

$$\phi = R\cos\left[u\left(t \pm \frac{z}{V}\right) + \psi\right] \quad \text{..................} \quad (1).$$

This may be interpreted as a train of waves of amplitude R, period $2\pi/u$, and phase ψ, travelling with velocity V. This train of waves is without beginning or end. Most of the results of physical optics have direct application to a disturbance of the above form.

No radiation is found in nature which has the properties of the above function. The fact alone that all natural radiations have beginning and end would suffice to render (1) an inadequate representation.

It is required to solve the problem how to represent any natural radiation faithfully without losing the conveniences connected with the form (1). The problem recalls the familiar process of harmonic analysis. This process is applicable only to periodic functions, whereas such a motion of the æther as constitutes white light is non-periodic. In this connection it has been pointed out by Gouy

that Fourier's theorem of double integrals enables us to express a wide class of functions in terms of circular functions. In fact, subject to certain limitations,

$$f(t) = \int_0^\infty (\mathrm{C} \cos ut + \mathrm{S} \sin ut)\, du,$$

where $\mathrm{C} = \frac{1}{\pi} \int_{-\infty}^{+\infty} f(v) \cos uv\, dv, \quad \mathrm{S} = \frac{1}{\pi} \int_{-\infty}^{+\infty} f(v) \sin uv\, dv.$

It is proved in the course of the present paper that this process is always legitimate when $f(t)$ is such a function of time as can occur in a physical problem.

The above theorem reduces $f(t)$ to the limit of a sum of simple circular function of time, the element being $du(\mathrm{C} \cos ut + \mathrm{S} \sin ut)$. If we write this $du \,.\, \mathrm{R} \cos (ut + \psi)$, a simple vibration of amplitude $\mathrm{R}du$, period $2\pi/u$ and phase ψ is suggested. To connect this analysis with the physical analysis of light into a continuous spectrum is tempting. The present essay is an attempt to prove that, in certain very general cases, such a connection exists. The proof depends upon the two principles (i) that we can have no cognizance of instants of time, but can observe only the contents of small intervals of time; (ii) that, in spectrum analysis, we do not deal with definite wave-lengths, but rather with small ranges of wave-length.

The fruitfulness of this calculus is illustrated by several applications. The radiation of an incandescent gas is discussed. The trains of waves emitted by molecules are continually being terminated by collisions. It is held that, in dealing with the limiting widths of spectrum lines, this effect must be included in the same investigation with the Doppler effect first pointed out by Lippich and Lord Rayleigh.

Other problems shown to be within the grasp of this method are: the connection between Röntgen rays as explained by Professor Thomson, and ordinary light; and the effect of radiative damping of the molecular vibrations in widening the lines of the spectrum. All these investigations are based upon a theorem for dealing with a radiation composed of a vast aggregate of similar pulses distributed at random. The theorem is due to Lord Rayleigh.

It is usual to examine the theory of dispersion by considering the action of a simple periodic force upon a simple vibrator. Since no light is simply periodic, it is necessary to extend the examination. This is done below. We have also inquired whether fluorescence can be due to natural vibrations of the molecules aroused by the non-periodic quality of light. It is shown that so long as the equation of motion is linear, no such explanation is possible.